explained how Daubechies used this Fourier transform to construct wavelets. The existence, uniqueness, and orthogonality of Daubechies wavelets is proved. It is also shown how other wavelets can be designed using the Fourier transform. Finally, Chapter 9 shows that wavelets can approximate signals accurately.

The author has indeed succeeded in writing a book for the intended audience. This is highly recommended as a textbook for an undergraduate course on (Fourier analysis and) wavelets.

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W. Freeden, T. Gervens, and M. Schreiner, *Constructive Approximation on the Sphere, with Application to Geomathematics*, Numerical Mathematics and Scientific Computation, Oxford University Press, 1998, xv + 227 pp.

Geomathematics is a rather recent discipline which deals with mathematics concerned with scientific problems from geochemistry, geodesy, geology, and geophysics. This books give a comprehensive treatment of spherical approximation in (geo)mathematics with emphasis on the theory of spherical harmonics and approximation by splines and wavelets. In the preface the authors say:

> *This book provides the necessary foundation for students interested in any of the diverse areas of constructive approximation in sphere-oriented geomathematics. It is designed as a graduate-level textbook and assumes some basic undergraduate training in linear algebra and (functional) analysis, plus some basic knowledge of numerical analysis. But the primary objective of the book is to help geophysicists and geo-engineers to understand future aspects of approximation by spherical harmonics and their modern counterparts (such as splines, wavelets, etc.).*

To achieve this goal, the textbook is divided into three parts, which are quite different in size. Part I, on *scalars*, of about 300 pages, contains 10 chapters; part II, on *vectors*, contains 34 pages and two chapters; part III, on *tensors*, has 45 pages and two chapters (4 pages for the last chapter). This difference in size can be explained by the observation that much of the theory of spherical harmonics, splines, and wavelets has a straightforward extension to vectors and tensors. Extensions of scalar results to the vectorial or tensorial case are discussed in more detail only if they are not straightforward. There is also a somewhat less distinctive division into a part dealing with the theory of spherical harmonics (Chapters 1–4 in the scalar case, Chapter 12 for vectors, and Chapter 14 for tensors), spherical spline theory (Chapters 5 and 6 for scalars, Chapter 13 for vectors, Chapter 15 for tensors), and spherical wavelet theory (Chapters 5, 7–11 for vectors, Chapter 13 for vectors, and Chapter 15 for tensors).

Most of the important properties of spherical harmonics can be described using homogeneous harmonic polynomials (Chapter 2). Spherical geometry is closely related to the orthogonal group $O(3)$ of real 3×3 orthogonal matrices and the special orthogonal group $SO(3)$ of orthogonal matrices with determinant 1. This approach is described in Chapter 3, where it is shown that spherical harmonics of given order form an irreducible invariant subspace of the space of square-integrable functions on the unit sphere. Spherical harmonics as eigenfunctions of the Beltrami operator, integral formulas, and the theory of Green's functions on the unit sphere are considered in Chapter 4.

Chapter 5 deals with spherical basis functions, with a development of general Sobolev spaces and invariant pseudodifferential operators which allows to recognize the radial basis function as an axisymmetric reproducing kernel function of the Sobolev space, which admits a Legendre expansion determined by the *spherical symbol* of the pseudodifferential operator. This chapter contains several examples of radial basis functions. Splines for spherical problems are the subject of Chapter 6, which gives a variational characterization of spline interpolation, error estimates, and convergence results.

The Gabor and Toeplitz transforms are described in Chapter 9, which is followed by the continuous wavelet transform in Chapter 10 and the discrete wavelet transform in Chapter 11. The basic tool is the theory of singular integrals on the sphere, which is given in Chapter 8.

Geoscientists and approximators will welcome this book as a fine exposition and collection of material which was otherwise only to be found in research journals. The bibliographical notes given at the end of each chapter allow interested researchers to find additional material.

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Proceedings

Advances in Multivariate Approximation, Werner Haussmann, Kurt Jetter, and Manfred Reimer, Eds., Mathematical Research **107**, Wiley–VCH, Berlin, 1999, 334 pp.

These are the proceedings of the 3rd international conference on Multivariate Approximation, which was held at Haus Bommerholtz, the guest house of the University of Dortmund, from September 27 through October 2, 1998. The book contains 22 refereed contributions taken from the 10 invited lectures and the 25 contributed talks of the scientific program. Several topics from modern aspects of multivariate approximation theory are covered, with emphasis on interpolation and approximation on spheres and balls, approximation by harmonic and subharmonic functions, multivariate splines, numerical integration, periodic interpolation, simultaneous approximation, and moduli of smoothness.

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